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ABSTRACT

The overidentification of structural equation models with latent variables is discussed. The use of two- and three-indicator models is not recommended since such models do not allow a testing of the crucial assumption of unidimensionality among indicators in most cases. Models with four or more indicators may be more sensitive to departures from unidimensionality among sets of indicators or an alleged common variable. Further, parsimonious models are to be preferred because they imply more observable consequences and more ways to be rejected, if false. Combining a parsimony index (James, Mulaik and Brett) with a normed fit index (Bentler and Bonnet) yields a normed parsimonious fit index. This combined index corrects for the tendency of the normed fit index to approach unity as the degrees of freedom of the model approach zero. Increasing the number of indicators, relative to the number of latent variables to be identified, can only serve to increase the parsimony of a model as long as the indicators are linked respectively to as few of the latent variables as possible. (PN)

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SOME COMMENTS ON THE OVERIDENTIFICATION OF MODELS IN STRUCTURAL EQUATION MODELING WITH LATENT VARIABLES

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Sometimes the things that we don't see, but which are right there in front of our faces, are more important than the things we do see. This I hold is especially true in the case of structural equation models.

To illustrate, consider the path diagram of a structural equation model with four manifest variables and two latent variables in Figure 1. This model is not only identified but is overidentified: With four observed variables there are four observed variances and six observed covariances among them; in other words, there are 10 observed coefficients. The number of parameters to estimate in this model is 9 structural coefficients. Solutions for these are given by Costner (1969). Thus with 10 observed coefficients and 9 to be estimated, that leaves one degree of freedom, and the model is overidentified.

Some researchers, I think, would be satisfied if they performed a chi-square goodness-of-fit test of this model against data involving estimated covariances among the observed variables and the test was not significant, yielding statistically acceptable fit. They might even think that in accepting the model they have established that the variables are connected by the causal paths described in the structural diagram. But the chi-square goodness-of-fit test in this case does not test hypotheses about the visible causal paths shown in the diagram between variables. (The tests concerning these causal paths must be performed with other statistics, e.g. confidence interval tests on the estimated parameters, or nested sequences of difference chi-square tests). Actually, the chi square test for goodness of fit concerns things that are not visible in the path diagram. These "invisible things" are the deleted pathways between variables of the model.

Figure 2 illustrates the model in Figure 1 with the deleted pathways indicated by broken lines. As one can see there are many deleted pathways. The deletion of these pathways is essential to the identification of this model. Furthermore, we are limited in what pathways we can put back into the model and still have an at least identified model. For example we cannot insert additional pathways between the latent exogenous or latent endogenous variables and the four indicators, for the coefficients associated with them would not be identified. With only one degree of freedom in our model we are at liberty to free up only one parameter (with our choice confined to covariances between indicators, respectively, of the two latent variables, as illustrated in Figure 3). Thus a goodness of fit test applied to our model in Figure 1 can only test the assumption that there is no correlated error term across indicators of different latent variables. That doesn't tell us much.

Costner (1969) and Costner and Schoenberg (1973) show us how using three indicator variables allows us to have more degrees of freedom and an ability to test more assumptions than we have with two-indicator models.

But they also point out that we are still limited with three-indicator models in detecting the presence of some forms of differential bias in our data. In particular, with three-variable models we are unable to detect the fact, especially in connection with a set of indicators of supposedly a single latent exogenous variable, that the hypothesis of a single latent variable is false. For example, consider the model in Figure 4 representing a three-indicator-per-latent-variable model, with two latent variables (aside from disturbances). This model would fit data perfectly that had been generated by the model in Figure 5, which has two latent variables determining the indicators of the exogenous variable of the model in Figure 4. The correlation between ξ_1 and ξ_2 need not be due to a single common latent variable, but to several, as in the model in Figure 6.

In Figure 6 the addition of a second indicator of ξ_1 would reveal the falsity of the assumption that there is a single common cause of the indicators $y_2,\ldots,y_4$ and that the endogenous variable η_1 is an effect of this common cause.

Similarly in three-indicator models we cannot detect the falsity of the assumption that the indicator variables are mediated by a single cause. For example, we may estimate the model in Figure 7, but it will fail to detect when the same data is generated by a model as in Figure 8. Including a fourth indicator that is an indicator of the second effect variable in Figure 8, would bring this to light.

In short, the use of two- and three- indicator models is not recommended. Such models do not allow us to test the crucial assumption of unidimensionality among our indicators in most cases. Models with four or more indicators may be more sensitive to departures from unidimensioality among sets of indicators of an alleged common variable.

There is another reason why we should use many (four or more) indicators for each latent construct variable in our models: parsimony. By parsimony is meant that the number of observed entities should be explained by many fewer entities. The philosopher Popper (1959), for example, stressed that we should put our models to the test by subjecting them to as many observable consequences as possible, and parsimonious models were to be preferred because they implied more observable consequences and more ways to be rejected, if false. He sought, furthermore, ways to index parsimony and met with partial success. In this vein I would like to offer the following index of parsimony for a model:

$$\Pi = \underline{m}/[\underline{p}(\underline{p} - 1)/2]$$

where $\underline{m}$ is the degrees of freedom for a model with $\underline{p}$ observed variables. What this represents is the ratio of the degrees of freedom of a model in question to the degrees of freedom of a null model postulating no relationship among the observed variables, which would be the model having potentially the worst possible fit to the observed variables. This null model would waste the fewest observed coefficients on estimation of parameters and use the greatest number of observed coefficients for testing the null model. Thus the null model is relatively the most parsimonious in

its use of the observations. Since a degree of freedom is lost in estimating each free, unspecified parameter of a model, our model with m parameters would waste some of its observed coefficients on estimating parameters. It would likely get better fit, but at the price of wasting observations on estimating parameters that otherwise would be useful in testing a model. So, the degrees of freedom of the null model is a maximum over linear models that might be associated with a given set of observed variables and can be used as a norm against which to compare the degrees of freedom of a given linear model associated with the observed variables.

I might add that in James, Mulaik and Brett (1982) we combined the parsimony index with Bentler and Bonnet's (1980) normed fit index to get a normed parsimonious fit index by simply multiplying the parsimony index times the normed fit index:

$$\pi = [m/p(p-1)/2]\,[(F_0 - F_k)/F_0]$$

where m and p are defined as before, F_0 is the index of (lack of) fit of the null model (of no relationship between variables), and F_k is the index of fit of the less restricted model, K, which contains the null model as a special case and has m degrees of freedom.

This normed parsimonious fit index corrects for the tendency of the normed fit index to approach unity as the degrees of freedom of the model approaches zero. A just-identified model with zero degrees of freedom and a normed fit index of unity would get a normed parsimonious fit index of zero.

To return to our main theme, increasing the number of indicators, relative to the number of latent variables to be indicated, can only serve to increase the parsimony of a model as long as the indicators are linked respectively to as few of the latent variables as possible. While I prefer the rule of "four or more" there may be other heuristics as well to follow, such as, "Use Ledermann's inequality or variants thereof to determine the minimum number of observed variables to determine a given number of latent construct variables" (cf. Thurstone, 1947, p. 293; Harman, 1960, p. 71; Mulaik, 1972, p. 138). But I think it is becoming clear that common practice in psychological research for testing hypotheses about hypothetical (latent) constructs has been quite inadequate because so few indicators are used in studies. There is no question that seeking to correct this deficiency will place a greater burden on us in requiring us to construct multiple indicators of a given hypothetical construct. But the payoff will be clearer indications where our hypotheses are in error.

References

Bentler, P. M. & Bonnett, D. G. Significance tests and goodness of fit in the analysis of covariance structures. Psychological Bulletin, 1980, 88, 588-606.

Costner, H. L. Theory, deduction, and rules of correspondence. American Journal of Sociology, 1969, 75, 245-263.

Costner, H. L. & Schoenberg, R. Diagnosing indicator ills in multiple indicator models. In Goldberger, A. S. & Duncan, O. D. (Eds.) Structural equation models in the social sciences. New York: Seminar Press, 1973.

Harman, H. Modern factor analysis. Chicago: University of Chicago Press, 1960.

James, L. R., Mulaik, S. A., & Brett, J. M. Causal analysis: assumptions, models, and data. Beverly Hills, California: Sage Publications, 1982.

Mulaik, S. A. The foundations of factor analysis. New York: McGraw-Hill, 1972.

Thurstone, L. L. Multiple-factor analysis. Chicago: University of Chicago Press, 1947.

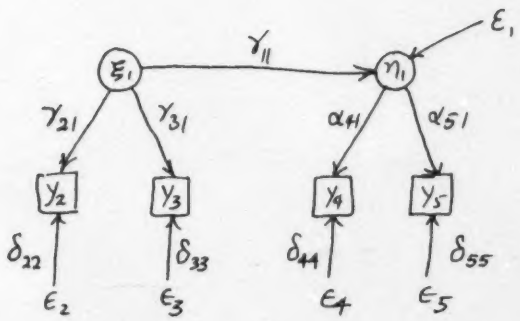

Figure 1. A two-indicator model.

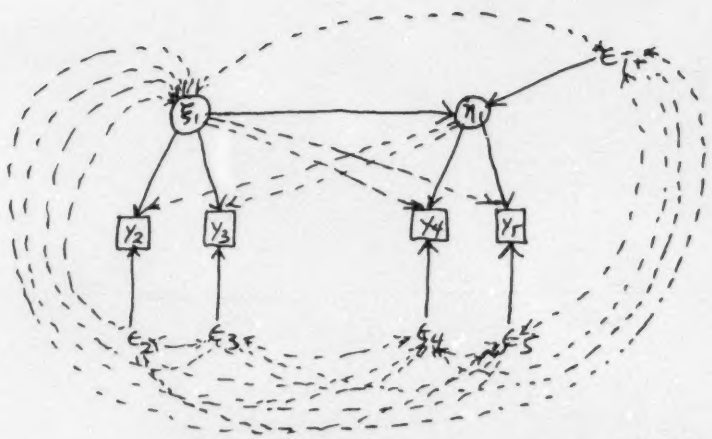

Figure 2. Deleted paths indicated by broken lines.

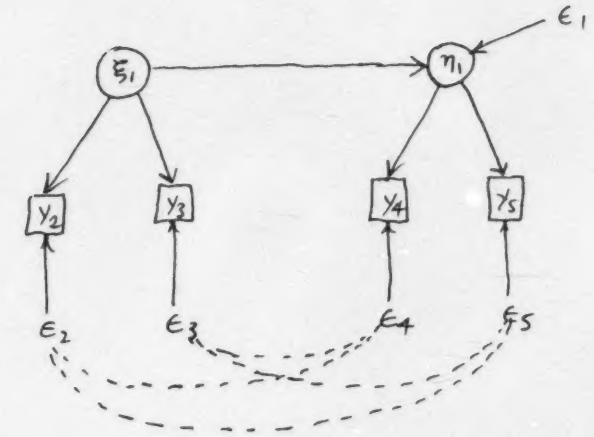

Figure 3. Potential paths that may introduce detectable differential bias.

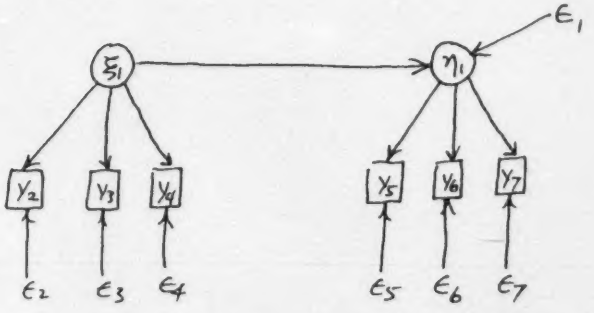

Figure 4. A three-indicator model.

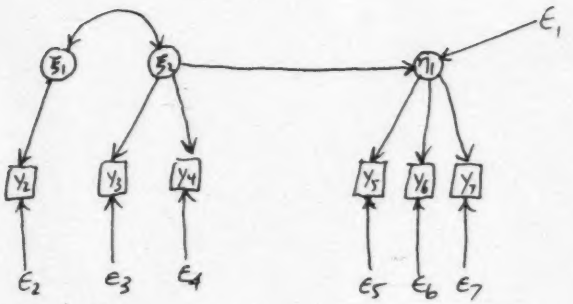

Figure 5. A model consistent with the model in Figure 4.

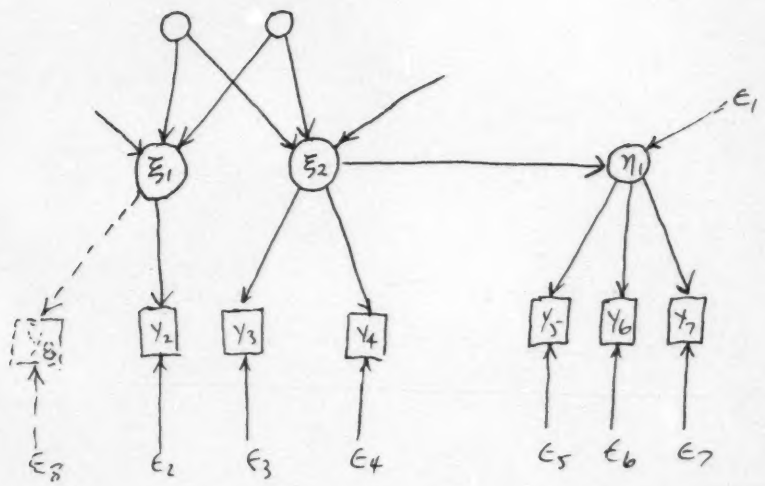

Figure 6. A model consistent with the model in Figure 4 until a fourth variable, y_8, is added.

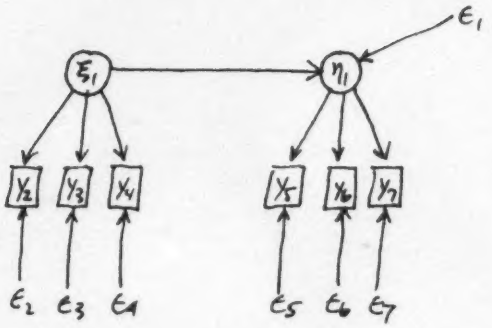

Figure 7 - A three indicator model indistinguishable from the model in Figure 8.

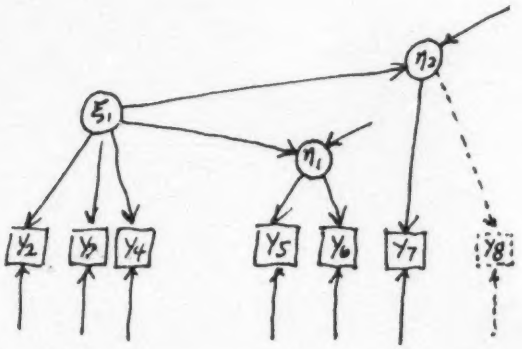

Figure 8. Model generating data consistent with model in Figure 7 until variable y_8 is added.